

# Quantum toy model for black-hole back-reaction

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We propose a simple quantum field theoretical toy model for black hole evaporation and study the back-reaction of Hawking radiation onto the classical background. It turns out that the horizon is also “pushed back” in this situation (i.e., the interior region shrinks) but this back-reaction is not caused by energy conservation but by momentum balance. The effective heat capacity and the induced entropy variation can have both signs – depending on the parameters of the model.

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*Introduction* Black holes are arguably the most simple and at the same time most intriguing objects in the universe. The no-hair theorem states that they can fully be described by a small set of parameters such as their mass  $M$  and angular momentum  $J$ . Yet our standard picture of black holes contains many striking properties: Even though black holes should be completely black classically, they emit Hawking radiation due to quantum effects [1]. This evaporation process causes the black hole (horizon) to shrink (in the absence of infalling matter) due to the back-reaction of Hawking radiation. Therefore, black holes possess a negative heat capacity [2], i.e., the temperature grows with decreasing energy. Extrapolating this picture till the final stages of the evaporation, the black hole should end up in an explosion, where its temperature blows up and thus effects of quantum gravity should become important. Perhaps most fascinating is the observation that the second law of thermodynamics apparently [3] requires to assign an entropy  $S$  to the black hole, which is determined by the horizon surface area  $A$  via  $S = A/4$  (in natural units  $\hbar = G = c = 1$ ).

Taking the analogy between black holes and thermodynamics seriously provides a very consistent picture, which has been confirmed by various gedanken experiments [3, 4] considering the construction of heat engines with black holes etc. It almost seems as if nature was trying to give us some hints regarding the underlying structure which unifies quantum theory and gravity – which we do not fully understand yet. In order to understand these hints better, it might be useful to ask the question of whether (and how) the aforementioned properties depend on the detailed structure of the Einstein equations or whether they are more universal. For example, the study of condensed-matter based black hole analogues [5, 6] shows that Hawking radiation is a fairly robust quantum phenomenon [7], which just requires the occurrence of an effective horizon and is quite independent of the Einstein equations. In contrast, the introduction of a black hole entropy with the desired properties seems to rely on the Einstein equations.

In the following, we try to further disentangle universal features from properties which are specific to black holes

(e.g., Einstein equations, rotational symmetry, conserved ADM mass). To this end, we propose a toy model which captures some of the relevant features of black holes and allows us to study the back-reaction of the emitted Hawking radiation onto the classical background solution.

*Toy Model* In the toy model we are going to discuss, the gravitational field will be represented by a real scalar field  $\psi$  in 1+1 dimensions with the Lagrangian ( $\hbar = 1$ )

$$\mathcal{L}_\psi = \frac{1}{2} \left( \dot{\psi}^2 - c_\psi^2 [\partial_x \psi]^2 \right) - V(\psi). \quad (1)$$

With respect to the propagation speed  $c_\psi$  of the  $\psi$  field, this form is Lorentz invariant. The potential  $V(\psi)$  is supposed to be very stiff, i.e., the field  $\psi$  is assumed to be heavy in the sense that it can be well approximated by a classical field. For definiteness, we choose the sine-Gordon potential  $V(\psi) \propto 1 - \cos(\psi/\psi_0)$ , but other potentials admitting stable solitonic solutions would also work. The global ground state  $\psi = 0$  then corresponds to a vanishing gravitational field whereas a kink (topological defect) models a black (or white) hole horizon

$$\psi(x) = -4\psi_0 \arctan(\exp\{-\xi[x - x_{\text{kink}}]\}) . \quad (2)$$

The position  $x = x_{\text{kink}}$  of the kink at rest is arbitrary and its width  $1/\xi$  is determined by  $V(\psi)$  and  $c_\psi$ . In comparison to other models of black holes (see, e.g., [8, 9]), the advantage of the above set-up lies in the topologically protected stability and localization of the kink, which behaves very similar to a particle (see also [10]).

In order to study Hawking radiation and its impact on the kink, we consider a massless quantum field  $\phi$  coupled to the heavy field  $\psi$  via the coupling constant  $g$

$$\mathcal{L}_\phi = \frac{1}{2} \left( [\partial_t \phi + g\psi \partial_x \phi]^2 - c_\phi^2 [\partial_x \phi]^2 \right) . \quad (3)$$

Note that the velocity  $c_\phi$  of the light (massless) field may differ from  $c_\psi$ . The propagation of the light field  $\phi$  in the approximately classical background  $\psi$  is completely analogous to that in a gravitational field described by the Painlevé-Gullstrand-Lemaître metric (cf. [5, 6])

$$ds^2 = (c_\phi^2 - v^2) dt^2 - 2v dt dx - dx^2, \quad (4)$$

where  $v = g\psi$  denotes the local velocity of freely falling frames. A horizon occurs if this velocity  $v$  exceeds the speed of light  $c_\phi$ . Based on the analogy to gravity, we may also derive the pseudo energy-momentum tensor of the  $\phi$  field with respect to the above metric  $g^{\mu\nu}$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{A}_\phi}{\delta g^{\mu\nu}} = (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} g_{\mu\nu} (\partial_\rho \phi)(\partial^\rho \phi). \quad (5)$$

The associated energy density  $T_0^0$  of the light field

$$\mathcal{H}_\phi = \frac{1}{2} ([\partial_t \phi]^2 + (c_\phi^2 - v^2) [\partial_x \phi]^2) \quad (6)$$

contains negative parts beyond the horizon  $v^2 > c_\phi^2$ . Of course, this is precisely the reason why effects like Hawking radiation are possible [11].

However, an energy density which is not bounded from below seems unphysical and typically indicates instabilities (already on the classical level). In order to avoid this problem, we may add an extra term which does not modify the linearized low-energy behavior of our model

$$\mathcal{L}_\phi^{\text{reg}} = \mathcal{L}_\phi - \alpha^2 (c_\phi^2 - v^2)^2 [\partial_x \phi]^4 - \frac{1}{16\alpha^2}, \quad (7)$$

but generates a positive definite energy density

$$\mathcal{H}_\phi^{\text{reg}} = \frac{1}{2} \left( [\partial_t \phi]^2 + \left[ \alpha(c_\phi^2 - v^2) [\partial_x \phi]^2 + \frac{1}{4\alpha} \right]^2 \right). \quad (8)$$

In the exterior region  $c_\phi^2 > v^2$ , the classical ground state is still given by  $\phi = 0$ , but beyond the horizon  $c_\phi^2 < v^2$ , we have  $2\alpha(\partial_x \phi) = (v^2 - c_\phi^2)^{-1/2}$ . Thus, the classical ground state profile would not be differentiable at the horizon, i.e., the term  $[\partial_x \phi]^2$  in the energy density, for example, would be ill-defined. This problem can be cured by adding another term (which again does not modify the low-energy behavior) and we finally arrive at the total Lagrangian of our toy model

$$\mathcal{L}_{\text{full}} = \mathcal{L}_\psi + \mathcal{L}_\phi^{\text{reg}} - \beta^2 [\partial_x^2 \phi]^2. \quad (9)$$

The last term smoothens the classical ground state profile at the horizon and induces a super-luminal dispersion relation  $(\omega + vk)^2 = c_\phi^2 k^2 + 2\beta^2 k^4$  at large wavenumbers.

*Back-reaction* The equation of motion of the light field can be derived from the Lagrangian above

$$(\partial_t + v\partial_x)(\partial_t + \partial_x v)\phi = c_\phi^2 \partial_x^2 \phi + \mathcal{O}(\partial_x^4), \quad (10)$$

where  $\mathcal{O}(\partial_x^4)$  denote the higher-order  $\alpha$  and  $\beta$  terms we added for stability and regularity reasons. Similarly, the heavy field evolves according to

$$\ddot{\psi} - c_\psi^2 \partial_x^2 \psi = V'(\psi) - g[\partial_t \phi + g\psi \partial_x \phi] \partial_x \phi + \mathcal{O}(\partial_x^4). \quad (11)$$

From the full set of equations, we see that the kink profile in Eq. (2) together with  $\phi = 0$  exactly solves the classical equations of motion (though it is not the ground

state). However, the impact of quantum fluctuations changes this picture: For  $2\pi g\psi_0 > c_\phi$ , the kink acts as a black hole horizon and thus emits Hawking radiation. Of course, the energy/momentum given off must come from somewhere and hence this quantum effects should have some impact on the classical kink background.

In order to estimate the quantum back-reaction, we quantize the fields  $\phi \rightarrow \hat{\phi}$  as well as  $\psi \rightarrow \hat{\psi}$  and employ a mean-field expansion  $\hat{\psi} = \psi_{\text{cl}} + \delta\hat{\psi}$  where  $\psi_{\text{cl}}$  denotes the classical kink profile in Eq. (2) and  $\delta\hat{\psi}$  as well as  $\hat{\phi}$  are supposed to be small (i.e.,  $\hat{\phi}, \delta\hat{\psi} \ll \psi_{\text{cl}}$ ). Taking the expectation value of Eq. (11) and comparing it with Eq. (5), we find that the lowest-order contributions of the quantum back-reaction force are just given by the expectation value of the pseudo energy-momentum tensor [12]

$$[\partial_t^2 - c_\psi^2 \partial_x^2 - V''(\psi_{\text{cl}})] \langle \delta\hat{\psi} \rangle \approx -g \langle \hat{T}_1^0 \rangle. \quad (12)$$

Remembering the covariant energy-momentum balance

$$\nabla_\mu T_\nu^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} T_\nu^\mu) - \frac{1}{2} T^{\alpha\beta} \partial_\nu g_{\alpha\beta} = 0, \quad (13)$$

we find that  $\langle \hat{T}_1^0 \rangle$  denotes the momentum density  $\pi_\phi \phi'$ , which varies with position in general. In contrast, the energy flux  $\langle \hat{T}_0^1 \rangle$  measured with respect to the stationary frame is constant  $\partial_x \langle \hat{T}_0^1 \rangle = 0$  for a kink at rest.

Fortunately, the expectation value  $\langle \hat{T}_\nu^\mu \rangle$  can be calculated analytically for a scalar field in 1+1 dimensions. In the Unruh state (which is the appropriate state for describing black-hole evaporation), one obtains [13]

$$\langle \hat{T}_1^0 \rangle = \frac{4vc_\phi(\kappa^2 - [v']^2 - \gamma vv'') - \kappa^2(c_\phi + v)^2}{48\pi c_\phi^3 \gamma^2}, \quad (14)$$

with  $\gamma = 1 - v^2/c_\phi^2$  and the effective surface gravity  $\kappa$  determining the Hawking temperature

$$T_{\text{Hawking}} = \frac{\kappa}{2\pi} = \frac{1}{2\pi} \left( \frac{dv}{dx} \right)_{v^2=c_\phi^2}. \quad (15)$$

Note that  $\langle \hat{T}_1^0 \rangle$  calculated in the Unruh state is regular across black-hole horizon  $v = -c_\phi$ , but singular at the white hole horizon  $v = +c_\phi$ . (The Israel-Hartle-Hawking state would be regular at both horizons.) Far away from the kink/horizon  $v \rightarrow 0$ , we just get the usual thermal flux  $\langle \hat{T}_1^0 \rangle = -\kappa^2/(48\pi c_\phi)$ .

The corrections induced by the quantum back-reaction can be visualized by incorporating them into an effective potential  $V_{\text{eff}}$  via

$$V'_{\text{eff}}(\psi) = V'(\psi_{\text{cl}}) - g \langle \hat{T}_1^0 \rangle. \quad (16)$$

For the classical potential  $V(\psi)$ , all minima  $\psi \in 2\pi\psi_0\mathbb{Z}$  occur at the same energy  $V = 0$ . However, the effective potential  $V_{\text{eff}}$  is distorted such that the central minimum is lower than the next one describing the black

hole interior  $V_{\text{eff}}(\psi = 0) < V_{\text{eff}}(-2\pi\psi_0)$ . In this sense, the exterior region is effectively energetically favorable and thus the horizon starts to move inwards, i.e., the black hole shrinks. Alternatively, the same result can be derived directly from Eq. (12) via classical time-dependent perturbation theory around the kink solution. The differential operator on the left-hand side of Eq. (12) possesses a continuum of gapped propagating (delocalized) modes with  $\omega^2 > 0$  and one localized zero-mode  $\propto 1/\cosh(\xi[x-x_{\text{kink}}])$  with  $\omega = 0$ , which just corresponds to a translation of the kink position [14]. After expanding the source term  $-g\langle\hat{T}_1^0\rangle$  in Eq. (12) into these modes, the perturbations in the continuous spectrum  $\omega^2 > 0$  just propagate away from the kink – whereas the spatial overlap between  $-g\langle\hat{T}_1^0\rangle$  and the zero-mode determines the acceleration  $\ddot{x}_{\text{kink}} < 0$  of the kink position.

*Energy and Momentum* In contrast to the fluid analogues for black holes (with a steady in- and out-flow of energy and momentum), for example, the kink considered here represents a well localized object, which allows us to ask the question of where the force pushing back the horizon comes from. In general, the contribution of the  $\phi$  field to the total energy-momentum tensor  $\partial_\mu T^{\mu\nu} = 0$  is different from the pseudo energy-momentum tensor  $\nabla_\mu T^{\mu\nu} = 0$  defined with respect to the effective metric (4), which complicates the analysis [15]. Fortunately, these difficulties are absent in our toy model where the mixed components of both tensors coincide  $T_\nu^\mu = T_\nu^\mu$ . The energy density  $T_0^0$  is given by Eq. (6) and the classical expression for the momentum flux density just reads  $T_1^1 = -T_0^0$  due to conformal invariance of the scalar field in 1+1 dimensions. Note, however, that the quantum expectation values differ due to the trace anomaly [13]. The energy flux density  $T_0^1 = \dot{\phi}\partial\mathcal{L}/\partial\phi'$  is given by  $T_0^1 = \dot{\phi}[v\dot{\phi} + (v^2 - c_\phi^2)\phi']$  and differs from the momentum density  $T_1^0$  in Eqs. (11) and (12) for  $v \neq 0$ .

Far away from the kink, we may estimate the above quantities by employing the geometric-optics approximation and replacing  $\phi \rightarrow \Omega$  and  $\phi' \rightarrow k$ . For solutions of the dispersion relation  $(\Omega + vk)^2 = c_\phi^2 k^2 + \mathcal{O}(k^4)$  corresponding to the outgoing Hawking radiation and its infalling partner particles, the energy density per normalized amplitude  $T_0^0 = c\Omega^2/(c - |v|)$  changes its sign at the horizon, cf. Eq. (6). The energy flux density  $T_0^1 = c\Omega^2$  is constant and positive everywhere (which is even true beyond the geometric-optics approximation). Note that  $\Omega$  is conserved as we are considering a quasi-stationary scenario. Thus, the total energy budget is balanced since the outgoing Hawking radiation carries away positive energy, but the infalling partners have a negative energy.

The momentum density  $T_1^0 = -c\Omega^2/(c - |v|)^2$ , on the other hand, turns out to be negative everywhere – or more precisely, far away from the kink, cf. the exact expression (14) with  $\Omega \sim \kappa$ . Thus the momentum flux density  $T_1^1 = -c\Omega^2/(c - |v|)$ , i.e., the pressure, also changes sign at the horizon. (The trace anomaly vanishes in

the asymptotic region  $v' = v'' = 0$  far away from the kink where the geometric-optics approximation applies  $T_1^1 = -T_0^0$ .) Consequently, while the Hawking particles carry away positive momentum and push back the kink, their infalling partner particles act in the opposite way and pull on the kink. In summary, the momentum is not balanced and thus the kink starts to move, i.e., the black-hole interior region shrinks.

*Thermodynamics* The application of thermodynamic concepts to our toy model (in analogy to real black holes) presents some difficulties and ambiguities: Considering the heat capacity  $C = dE/dT$ , for example, we would associate  $T$  with the Hawking temperature (15). The variation of the internal energy  $dE$ , however, could be identified with the heat given off by the Hawking radiation  $dE = \delta Q \propto \kappa^2 dt$  or with the change of the kinetic energy of the kink  $E = M_{\text{eff}}\dot{x}_{\text{kink}}^2/2$  (for  $\dot{x}_{\text{kink}}^2 \ll c_\psi^2$ ). Since the kink does not possess a conserved ADM mass, these quantities will be different in general. Either way, the heat capacity  $C = dE/dT$  could be positive as well as negative since the Hawking temperature can be increased  $dT > 0$  or decreased  $dT < 0$  by the quantum back-reaction of the evaporation process. There are several different effects: Due to the distortion of the effective potential  $V_{\text{eff}}$ , the shape of the kink deviates from the classical profile (2). This deviation is governed by the aforementioned continuum modes  $\omega^2 > 0$ . Furthermore, the kink starts to move – which is described by the zero-mode. The motion of the kink, in turn, implies a Doppler shift of the Hawking radiation. Finally, even in the rest frame of the kink, the position of the horizon  $x_h$  changes since the kink velocity  $\dot{x}_{\text{kink}}$  effectively reduces the local frame-dragging speed  $v$  and therefore the surface gravity  $\kappa = v'(x_h)$  may change. As a result of all these effects, the heat capacity depends on many parameters ( $c_\phi$ ,  $c_\psi$ , and  $g\psi_0$  etc.) and may assume negative as well as positive values. In order to demonstrate this sign ambiguity, let us consider the case  $c_\psi \gg c_\phi$  for simplicity. In this limit, the continuum modes  $\omega^2 > 0$  are very fast and hence the change of the shape of the kink can be neglected, i.e., the quantum back-reaction induces a rigid motion of the kink only. As another simplification, the transformation of the  $\psi$ -field into the rest frame of the kink is just a Galilei transformation due to  $c_\psi \gg c_\phi$ . The new horizon position is then simply determined by  $v(x_h) = -c_\phi + \dot{x}_{\text{kink}}$ . Linearizing this equality together with  $\kappa = v'(x_h)$ , we find that the variation  $\delta\kappa$  of the surface gravity induced by the acceleration of the kink  $\delta\dot{x}_{\text{kink}}$  is determined by  $\delta\kappa = v''(x_h)\delta\dot{x}_{\text{kink}}/\kappa$ . Since  $v''(x_h)$  can be positive or negative (depending on the relation between  $c_\phi$  and  $g\psi_0$ ), the temperature measured in the rest frame of the kink could change in both directions. The temperature in the laboratory frame acquires an additional Doppler shift, which is given by  $\delta\kappa = -\kappa\delta\dot{x}_{\text{kink}}/c_\phi$ . The relative strength of the two competing effects (Doppler shift and horizon displacement)

is given by  $c_\phi v''/(v')^2$ , which can be above or below one. Ergo, both temperatures (in the kink frame and in the laboratory frame) may increase or decrease due to the back-reaction of Hawking radiation, i.e., the heat capacity can be positive or negative (or even infinite – at the turning point where  $\delta T = 0$ ).

Similar ambiguities apply to the entropy  $dS = dE/T$ . Choosing  $dE = \delta Q \propto \kappa^2 dt$  just reproduces the entropy balance of the Hawking radiation in the exterior region – which is of course indeed thermal. Inserting the kinetic energy  $E = M_{\text{eff}} \dot{x}_{\text{kink}}^2/2$ , on the other hand, we could violate the 2<sup>nd</sup> law since the kink can be slowed down by incident coherent radiation (carrying zero entropy).

**Conclusions** Modeling the black hole (horizon) by a stable topological defect in the form of a kink, we were able to derive the quantum back-reaction of the resulting evaporation process. It turns out that the kink/horizon is also pushed inwards as in a real black hole but, in contrast to the gravitational case, this back-reaction force is not caused by energy conservation but by momentum balance. Energetically, the expansion of the horizon would be favorable because the minimum energy density in exterior region  $\phi = \psi = 0$  is far above  $1/(4\alpha)^2 > 0$  the ground state in the interior region. Hence, going beyond the linear analysis performed here, one might suspect that the  $\phi$  field approaches its ground state via non-linear (quantum) instabilities until the evaporation stops.

Further thermodynamical concepts such as heat capacity or entropy (variation) cannot be defined unambiguously and can have both signs – depending on the considered parameters [16]. Together with the results in [9], our calculations and the energy-momentum considerations above suggest that Hawking radiation and the resulting back-reaction force “pushing” the horizon inwards may be universal – whereas the heat capacity and the entropy concept strongly depend on the underlying structure (e.g., Einstein equations). Note that in the Israel-Hartle-Hawking state with the expectation value being  $\langle \hat{T}_1^0 \rangle = v(\kappa^2 - [v']^2 - \gamma v v'')/(12\pi c_\phi^2 \gamma^2)$ , the horizon is still pushed inwards – i.e., it does not correspond to the thermal equilibrium state for the combined system [kink in Eq. (2) plus  $\phi$  field].

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- [2] For simplicity, we are considering black holes without charge  $Q = 0$  and angular momentum  $J = 0$ . Otherwise we would have to distinguish different heat capacities in analogy to  $c_p$  and  $c_v$  in thermodynamics, as can be seen by comparing the first law of thermodynamics  $dE = TdS - pdV$  with the first law of black hole dynamics  $dM = \kappa dA/(8\pi) + \Omega_H dJ + \Phi dQ$  where  $\Omega_H$  is the angular velocity (at the horizon) and  $\Phi$  is the (electrostatic) potential.
- [3] J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. **31**, 161 (1973); J. D. Bekenstein, Lett. Nuovo Cim. **4**, 737 (1972); Phys. Rev. D **7**, 2333 (1973); ibid. **9**, 3292 (1974); ibid. **12**, 3077 (1975).
- [4] See, e.g., W. G. Unruh and R. M. Wald, Phys. Rev. D **25**, 942 (1982).
- [5] W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981).
- [6] C. Barceló, S. Liberati, and M. Visser, Living Rev. Rel. **8**, 12 (2005); and references therein.
- [7] See, e.g., W. G. Unruh and R. Schützhold, Phys. Rev. D **71**, 024028 (2005); and references therein.
- [8] See, e.g., O. B. Zaslavskii, Class. Quant. Grav. **20**, 2963 (2003); S. Nojiri and S. D. Odintsov, Phys. Rev. D **59**, 044003 (1999); and references therein.
- [9] R. Balbinot, S. Fagnocchi, A. Fabbri and G. P. Procopio, Phys. Rev. Lett. **94**, 161302 (2005); R. Balbinot, S. Fagnocchi and A. Fabbri, Phys. Rev. D **71**, 064019 (2005).
- [10] Ted Jacobson and Tatsuhiko Koike, *Black hole and baby universe in a thin film of  $^3\text{He-A}$* , in M. Novello, M. Visser, and G. Volovik (editors), *Artificial Black Holes* (World Scientific, Singapore, 2002); [arXiv:cond-mat/0205174](#).
- [11] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, (Cambridge University Press, Cambridge, England 1982).
- [12] In a similar way, one could calculate the back-reaction of the quantum fluctuations of the  $\psi$  field itself. However, symmetry arguments show that these corrections do not generate any force on the kink.
- [13] P. C. Davies, S. A. Fulling and W. G. Unruh, Phys. Rev. D **13**, 2720 (1976).
- [14] A. R. Bishop, J. A. Krumhansl, S. E. Trullinger, Physica D **1**, 1 (1980); M. B. Fogel, S. E. Trullinger, A. R. Bishop, J. A. Krumhansl, Phys. Rev. B **15**, 1578 (1977); J. Rubinstein, J. Math. Phys. **11**, 258 (1970).
- [15] M. Stone, Phys. Rev. E **62**, 1341 (2000); Phys. Rev. B **61**, 11780 (2000); *Phonons and forces: Momentum versus pseudomomentum in moving fluids*, in M. Novello, M. Visser, and G. Volovik (editors), *Artificial Black Holes* (World Scientific, Singapore, 2002); [arXiv:cond-mat/0012316](#).
- [16] The fate of the evaporating kink at late times also depends strongly on the parameters. E.g., for  $c_\psi \ll c_\phi$ , the kink will behave like a massive relativistic particle under the influence of an accelerating force. I.e., it will be accelerated towards  $|\dot{x}_{\text{kink}}| \uparrow c_\psi$  while its slope becomes constantly steeper due to the Lorentz contraction. As a result, the surface gravity grows – accelerating the kink even more. This mechanism will result in a *blow-up* where eventually the higher-order terms will become important.